* If the system contains two variables, AND the equ in the system are easy to graph, then graph them.
* It is imperative that you check all solutions, b/c extraneous solutions can creep in when solving nonlinear systems

WS#10-6
Systems of Non-linear Equations

1. Solve by indicated method and name each figure:

A. Substitution

$$\begin{cases} 3x - y = -2 \\ 2x^2 - y = 0 \end{cases}$$

$\left(-\frac{1}{2}, \frac{1}{2}\right)$
$(2, 8)$

(1) $y = 3x + 2$

(2) $2x^2 - (3x + 2) = 0$

↳ $2x^2 - 3x - 2 = 0$

(3) Factor + solve for x

$(2x+1)(x-2) = 0$

$x = -\frac{1}{2}$, $x = 2$

(4) Sub these values in for x in the function

$y = 3x + 2$

↳ $y = 3\left(-\frac{1}{2}\right) + 2 = \frac{1}{2}$

↳ $y = 3(2) + 2 = 8$

(5) The apparent solutions are $\left(-\frac{1}{2}, \frac{1}{2}\right)$ & $(2, 8)$

(6) Check.

$3\left(-\frac{1}{2}\right) - \left(\frac{1}{2}\right) = -2$ ✓ $2\left(-\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) = 0$ ✓ $> \left(\frac{1}{2}, -\frac{1}{2}\right)$

$3(2) - 8 = -2$ ✓ $2(2)^2 - 8 = 0$ ✓ $> (2, 8)$

B. Elimination

$$\begin{cases} x^2 + y^2 = 13 \\ x^2 - y = 7 \end{cases}$$

$(2, -3), (-2, -3)$
$(3, 2), (-3, 2)$

(1) graph each equ to see there are 4 solutions

(2) Subtract, then factor

$x^2 + y^2 + 0y = 13$
$-x^2 + 0y^2 - y = 7$

$y^2 + y = 6$
$y^2 + y - 6 = 0$
$(y+3)(y-2) = 0$

$y = -3$, $y = 2$

(3) use these in the 2nd equ to find x.

- $x^2 - (-3) = 7 \rightarrow x^2 = 4$
  $x = \pm 2$
- $x^2 - 2 = 7 \rightarrow x^2 = 9$
  $x = \pm 3$

(4) Verify that $(2, -3)$, $(-2, -3)$, $(3, 2)$, & $(-3, 2)$ are each solutions

C.

$$\begin{cases} x^2 + x + y^2 - 3y + 2 = 0 \\ x + 1 + \dfrac{y^2 - y}{x} = 0 \end{cases}$$

$x = -1, y = 1$

(1) tough to graph, so go right to the elimination method

↳ mult. 2nd equ by $(-x)$ AND add

$x^2 + x + y^2 - 3y + 2 = 0$
$+ -x^2 - x - y^2 + y + 0 = 0$

$-2y + 2 = 0 \rightarrow y = 1$

(2) Sub $y = 1$ into 1st equ & solve

$\rightarrow x^2 + x + (1)^2 - 3(1) + 2 = 0$

$x^2 + x = 0 \rightarrow x(x+1) = 0$

$x = 0$, $x = -1$

- x can't be zero b/c in the 2nd equ you would be dividing by 0.

(3) Check $x = -1$, $y = 1$

$(-1)^2 + (-1) + (1)^2 - 3(1) + 2 = 0$ ✓

$(-1) + 1 + \frac{(1)^2 - 1}{-1} = 0$ ✓

D. **Any method**

$\begin{cases} x^2 - y^2 = 4 \\ y = x^2 \end{cases}$

NRS

inconsistent system

(1) replace x^2 by y in 1st equ.

$y - y^2 = 4$

(2) rewrite

$y^2 - y + 4 = 0$

(3) can't factor, so find discriminant

$(-1)^2 - 4(1)(4) = -15$

(4) since discriminant < 0, the system has no real solutions

E. $\begin{cases} 3xy - 2y^2 = -2 \\ 9x^2 + 4y^2 = 10 \end{cases}$

$\left(\frac{\sqrt{2}}{3}, \sqrt{2}\right), \left(\frac{-\sqrt{2}}{3}, -\sqrt{2}\right)$

$(1, -\frac{1}{2}), (-1, \frac{1}{2})$

(1) mult 1st equ by 2 and add to 2nd equ

$6xy - 4y^2 = 4$

$+ 9x^2 + 4y^2 = 10$

$9x^2 + 6xy = 6$

$\frac{6xy}{6x} = \frac{6 - 9x^2}{6x} \rightarrow y = \frac{2 - 3x^2}{2x}, x \neq 0$

(2) Sub for y into 2nd equ

$9x^2 + 4\left(\frac{2 - 3x^2}{2x}\right)^2 = 10$

(3) Plug x-values into $y = \frac{2 - 3x^2}{x}$ to get y-values

$9x^2 + \frac{4 - 12x^2 + 9x^4}{x^2} = 10$

$9x^4 + 4 - 12x^2 + 9x^4 = 10x^2$

$9x^4 - 11x^2 + 2 = 0$

$(9x^2 - 2)(x^2 - 1) = 0$

$9x^2 - 2 = 0 \quad x^2 - 1 = 0$

$\frac{9x^2}{9} = \frac{2}{9} \quad x^2 = 1$

$x = \pm\frac{\sqrt{2}}{3} \quad x = \pm 1$

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2. The sum of two numbers is 7 and the difference of their squares is 21. Find the numbers.

$\begin{cases} x + y = 7 \\ x^2 - y^2 = 21 \end{cases}$

(1) From 1st equ, solve for y

$y = -x + 7$

(2) Sub into 2nd equ + solve

$x^2 - (-x + 7)^2 = 21$

$x^2 - (x^2 - 14x + 49) = 21$

$14x - 49 = 21$

$\frac{14x}{14} = \frac{70}{14} \rightarrow x = 5$

(3) Sub into 1st equ + solve

$(5) + y = 7 \rightarrow y = 2$

(4) Check $x = 5, y = 2$

$(5) + (2) = 7$ ✓

$(5)^2 - (2)^2 = 21$ ✓

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3. An area of 52 square feet is to be enclosed by two squares whose sides are in the ratio of 2:3. Find the sides of the squares.

$2x$ □ $3x$ □

(1) $(2x)^2 + (3x)^2 = 52$

$4x^2 + 9x^2 = 52$

$\frac{13x^2}{13} = \frac{52}{13} \rightarrow x^2 = 4$

$x = \pm 2 \rightarrow$ can only be $x = 2$

(2) Sides of squares:

Sub in $x = 2$

↳ $2(2) = 4$ Ft

$3(2) = 6$ Ft

4 Ft x 6 Ft